

DETERMINATION OF STRUCTURAL MODES OF VIBRATION USING DIGITAL PHOTOGRAMMETRY

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ABSTRACT

In this paper it is shown that it is possible to recover the three-dimensional modes of vibration of an oscillating structure through an off-line digital photogrammetric approach employing only a single video camera. In addition to the camera, the technique requires a high powered synchronised strobe unit and the careful control of fixed sequential delays between the excitation force, the strobe illumination and the multiple sequence of camera exposures. The dynamic 3D object point triangulation problem is then reduced to a set of static problems by capturing object shape in a number of different phases at a given number of measurement epochs. For each repeatable oscillation cycle, the vibrating object is imaged from a different camera position. The *XYZ* object coordinates of the target array at each sampling epoch can thus be determined by photogrammetric triangulation using all images corresponding to the same instant of time within the oscillation cycle. The dynamic mode shape is then determined from the triangulated object points at the various phases. The process is illustrated for the determination of modes of vibration for an aircraft wing section.

1. INTRODUCTION

A fundamental operation in structural dynamics is the determination, either experimentally or theoretically, of both mode shapes and their corresponding frequencies. These modes, of which there are an infinite number, can be shown to be orthogonal to one another when weighted by the mass density. The modes essentially summarise both the inertia and elastic behaviour of the structure and as a result form a complete set of functions a finite number of which can efficiently represent the “low” frequency motion of the structure in question.^{1,2} A modal expansion is also quite commonly referred to in control systems literature for Multi-Input Multi-Output systems and is normally expressed as a state space model.³ Experimental determination of mode shapes is of fundamental importance for the analysis of structural dynamics in the aircraft industry, where they are used to check on the validity of, or to improve, finite element models.⁴ These models are in turn used to determine the all-important aircraft flutter (stability) boundaries.

The present methods of measuring mode shapes typically require either a single accelerometer which is moved from point to point, or alternatively a large number of accelerometers which are placed all over the aircraft. The former method is cheap but

time consuming, whereas the latter is much faster, due to parallel processing, but also significantly more expensive. The method employing multiple accelerometers also suffers from the disadvantage of added mass due to the presence of instrumentation and wiring. A prospective alternative to accelerometer-based approaches, which has been adopted for the reported project, is vision-based optical 3D coordinate measurement via digital close-range photogrammetry. This technique, which is also termed vision metrology or videogrammetry, offers a practical, inexpensive, non-interfering method of measuring modes.

2. OFF-LINE AND ON-LINE VISION METROLOGY

Prior to discussing the novel application of vision metrology for the measurement of mode shapes, it is first useful to offer a brief explanation of the basic principles of this technique. Figure 1 depicts a so-called monoscopic/convergent close-range photogrammetric network in which a targetted object is imaged from four different camera station positions. The term convergent describes the nature of the orientation of the camera optical axes for the four image stations. This is distinct from stereo photogrammetry where the optical axes are constrained to be near parallel so as to allow human stereo viewing. Through a spatial intersection process, the XYZ coordinates of signalised points on the object are determined from both measured xy frame coordinates for the corresponding image points, and from knowledge of the exterior orientation (3D position and spatial orientation) of each image. The basic mathematical model for this transformation is given by the well-known collinearity equations^{5,6}

$$\begin{aligned} x - x_0 + \Delta x &= -c \frac{X'}{Z'} \\ y - y_0 + \Delta y &= -c \frac{Y'}{Z'} \end{aligned} \quad (1)$$

where

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \mathbf{R} \begin{pmatrix} X - X^0 \\ Y - Y^0 \\ Z - Z^0 \end{pmatrix}$$

Here, X', Y', Z' represents a rotated object space coordinate system with its origin at the camera perspective centre and its axes aligned to those of the image coordinate system xyz .

The collinearity equations describe the perspective transformation between the object space (object point X, Y, Z and perspective centre X^0, Y^0, Z^0 with rotation matrix \mathbf{R}) and image space (image point x, y) for a image recorded with a camera focal length c . Equation 1 also comprises sensor system calibration terms, namely the so-called interior orientation or intrinsic parameters, x_0, y_0 and c , and the image coordinate perturbation terms Δx and Δy which account for the departures from collinearity due mainly to lens distortion.

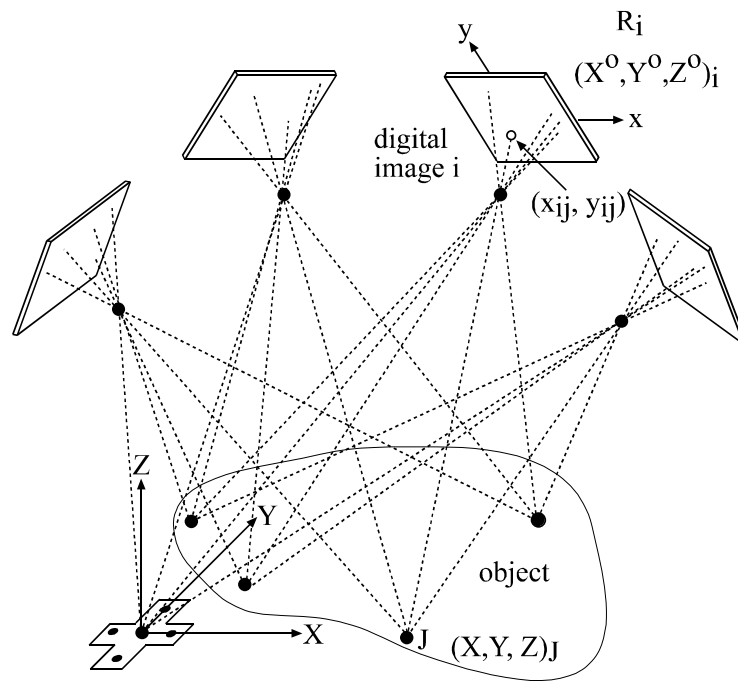


Figure 1: Convergent close-range photogrammetric network comprising four camera stations.

Any photogrammetric network comprising two or more camera stations and more than five object points common to each image constitutes an overdetermined non-linear observation equation system, the parameters for which can be obtained via a least-squares estimation process. Specifically, upon linearization Eqs. (1) can be written as

$$\mathbf{v} = \mathbf{A}_1 \mathbf{x}_1 + \mathbf{A}_2 \mathbf{x}_2 + \mathbf{A}_3 \mathbf{x}_3 + \mathbf{w} \quad (2)$$

Here \mathbf{x}_1 represents the parameters of camera position and orientation, \mathbf{x}_2 the object point coordinates and \mathbf{x}_3 the camera calibration parameters. The \mathbf{A}_i matrices are the corresponding configuration matrices, \mathbf{w} is the image coordinate discrepancy vector, and \mathbf{v} is the vector of residuals of the observed image coordinates. The least-squares solution process for Eqs 2, which minimises the quadratic form $\mathbf{v}^T \mathbf{v}$, is termed the bundle adjustment.^{5,6} Although it is possible to solve simultaneously for all parameters \mathbf{x}_i within a single multi-station convergent imaging network, special attention must be paid to network geometry if a sensor ‘self-calibration’ is also to be achieved. For the present discussion we will disregard this aspect and concentrate solely on the recovery of the object point coordinates (\mathbf{x}_2) and sensor exterior orientation (\mathbf{x}_1).

Within vision metrology there are two distinct approaches that can be followed, namely off-line and on-line (real-time) configurations. In the case of an off-line network, the images indicated in Figure 1 would generally be recorded sequentially with a single camera. Following acquisition of all images, of which there may be 3, 10 or even 100, image coordinates are observed and the solution for the parameters \mathbf{x}_1 and \mathbf{x}_2 is obtained via least-squares normal equations formed from Eqs. 2. This means

that in addition to the object coordinates (X_j, Y_j, Z_j) for each target j , the exterior orientation parameters (X^0, Y^0, Z^0) with the three angles forming the rotation matrix R for all camera stations are simultaneously recovered even though they are not explicitly sought within the measuring process.

Off-line vision metrology is widely used for high-precision measurement with the sectors of large-scale manufacturing and engineering.⁷ Innovations such as orientation devices and coded targets, along with 'intelligent' CCD cameras, can be incorporated to enable the off-line measurement process to be conducted fully automatically.^{8,9} The automated process can be briefly summarised as follows: Let us first imagine that the targetted cross in Figure 1 comprises four points of known XYZ coordinates (i.e. known relative to one another in an arbitrary local cartesian reference system), which are automatically identifiable within each digital image. Following the automatic measurement of all image point coordinates (x,y) , an initial solution for sensor exterior orientation can be obtained on an image-by-image basis via a spatial resection from the known coordinates of the orientation device:

$$\mathbf{v} = A_{1i} \mathbf{x}_{1i} + \mathbf{w}_i \quad (3)$$

A least-squares approach is again typically adopted for the solution of the six parameters forming \mathbf{x}_{1i} . Once the preliminary exterior orientation is established, approximate object point coordinates can be computed, again sequentially for each point j , through a series of space intersections in three unknowns. The observation equations follow from Eq. 2 as

$$\mathbf{v} = A_{2j} \mathbf{x}_{2j} + \mathbf{w}_j \quad (4)$$

The off-line approach to digital close-range photogrammetry is well documented in the literature as it represents the traditional approach to the restitution of convergent photogrammetric imaging configurations. A general prerequisite for the process is that the shape of the object is invariant throughout the period when the imagery is recorded. The object may of course move during the recording period, but there can be no shape deformation of the target point field. At first sight, then, it would seem that a single-sensor, off-line vision metrology approach could not be used to determine the modes of vibration of a shape-variant, oscillating structure.

The second approach to vision metrology, through use of an on-line (real-time) system configuration, utilises multiple cameras which synchronously record the necessary imagery. Thus, in a real-time system, the camera stations in Figure 1 would each be occupied by a different camera. This configuration is much better suited to dynamic events since the object is recorded at each measurement epoch in an instant of time. The data processing can follow the same sequence as in the off-line approach, namely exterior orientation followed by spatial intersection and then bundle adjustment, though it is quite common to perform a precise initial exterior orientation followed at each measuring epoch by an intersection alone (Eq. 4).

Thus, if it is desired to track the motion of the object in Figure 1, a bundle adjustment can first be carried out to achieve an optimal exterior orientation. The camera station parameters are then held fixed, reflecting the stable nature of the multi-camera configuration, and at each subsequent recording epoch, the new positions of object points are determined by triangulation, always within a common *XYZ* reference system. On-line vision metrology systems are also widely employed in industrial measurement, with the object being ‘tracked’ usually comprising a targetted touch probe.⁹ This gives rise to an optically based real-time 3D coordinate measurement system. The technique of multi-camera, on-line digital photogrammetry has recently been employed by NASA to study vibrations of the large solar arrays on the Hubble Space Telescope.¹⁰ Stereo photogrammetry (using two cameras) has also been tried to study vibrations of models in wind tunnels.¹¹

Based on this cursory introduction alone, one could well surmise that the multi-camera, on-line mode of operation would be logical for the task of monitoring a vibrating structure. Yet, one of the distinctive features of the reported investigation is that an off-line, single-sensor vision metrology approach was successfully employed to determine spatial position information on a shape-variant vibrating structure.

3. OFF-LINE MEASUREMENT OF THE SHAPE-VARIANT WING

Shown in Figure 2 is a targetted wing section in a supporting frame, the wing being subjected to vibration forces. It is desired to measure the array of surface points at various instances within the cycle of oscillation. As mentioned above, this would logically be approached using an on-line, multi-sensor vision metrology approach. In order to employ single-sensor, off-line measurement, the dynamic problem must be reduced to a series of static single-sensor, multi-station measurement networks. To achieve this aim, a number of prerequisites must be met:

- The motion of the oscillating body must be completely reproducible.
- The motion must be phased-locked to a driving reference signal.
- The recording epochs for the imagery must also be phased-locked to that same reference signal.

With these conditions fulfilled it could be assumed that the shape of the object, the deformed wing section in this case, would be the same at a given instant of time within the repeatable cycle of oscillation, which is induced via a controlled electromagnetic shaker.

A modified version of the *Australis* software suite for off-line digital close-range photogrammetry¹² was to be employed for the image mensuration and photogrammetric data processing phase of the project. In its standard operational mode, *Australis* performs the automated measurement of each of the images forming the imaging network (Figure 1). Figure 3 shows a series of labelled targets, both wing section targets subject to vibration, and a small number (single and double-digit labels) of stable reference points. Once the *xy* image coordinates of each image forming the network for a given epoch are recorded, the spatial intersection to yield *XYZ* object target coordinates can be performed, again within the *Australis* system.

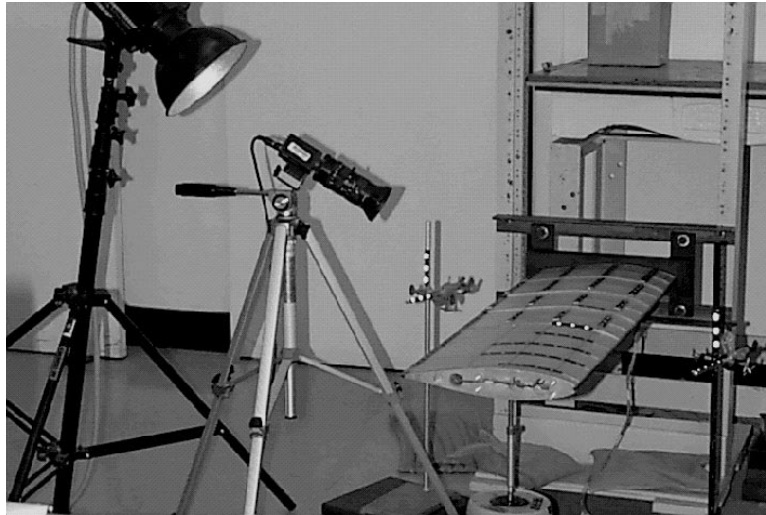


Figure 2: Imaging configuration for the targeted, vibrating wing section; eight such camera positions are employed.

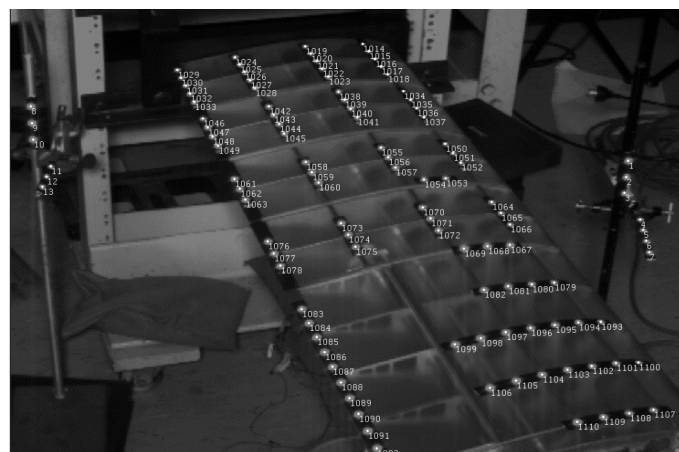


Figure 3: Object point array of retroreflective photogrammetric targets on the wing section and off-wing stable targets.

For the measurement of modes of vibration of the wing section, a variation to this computational procedure was employed. Given that a sequence of 20 or more images was taken from a single camera station position before the camera was moved to the next station and the shake/vibration sequence was repeated, it was deemed more practical to measure all images from each camera station sequentially. The maximum amplitude of vibration was 2 mm and thus at a imaging scale of approximately 1:100, the position of each target within successive images was repeatable to within about 5 pixels. This meant that following semi-automatic measurement of the first image of a sequence, which could be at the 'rest' position, all remaining xy coordinates could be measured with only a minimum of operator intervention. At a given recording epoch, the stable points would retain their labels, but the moving targets would be assigned labels where the first digit defined the frequency of oscillation, the second digit defined the delay or phase and the last three digits formed the constant location identifier.

This highly automated image measurement operation allowed the very rapid measurement of the tens of images forming the repeated 8-station triangulation networks. Moreover, the process provided a means to ensure that any instability in the camera exterior orientation could be compensated. A first stage of the computation process was to perform an 32-image triangulation for each frequency. A second and final stage was to compute a bundle adjustment which facilitated a simultaneous exterior orientation and triangulation for all $8i$ images and $(110i + 12)$ object points, where i is the number of frequencies multiplied by the number of delays. For the nominal $i = 8$, the final bundle adjustment comprised 384 exterior orientation parameters and about 900 object point coordinate parameters.

An important by-product of the spatial intersections, and especially the final bundle adjustment, is covariance information related to the XYZ target point coordinates. The resulting standard errors of triangulation for points on the wing surface were close to 0.05 mm, which was consistent with design expectations. For a vibration test using 64 images, the total image measurement and triangulation phase consumed only a few minutes. Within the repeatable cycles of oscillation it was desired to capture positional information at 97 targetted wing locations and at four phases of motion. The video imagery recorded at each station covered several cycles of oscillation and optimal images for measurement were chosen on the basis of target illumination. The time delay between the camera exposure and strobe illumination was varied to optimise the contrast between targets and background. An ideal intensity distribution was one which was bimodal, as indicated in Figure 4, where the spike at a greyscale value of 255 represents the return from the retroreflective targets.

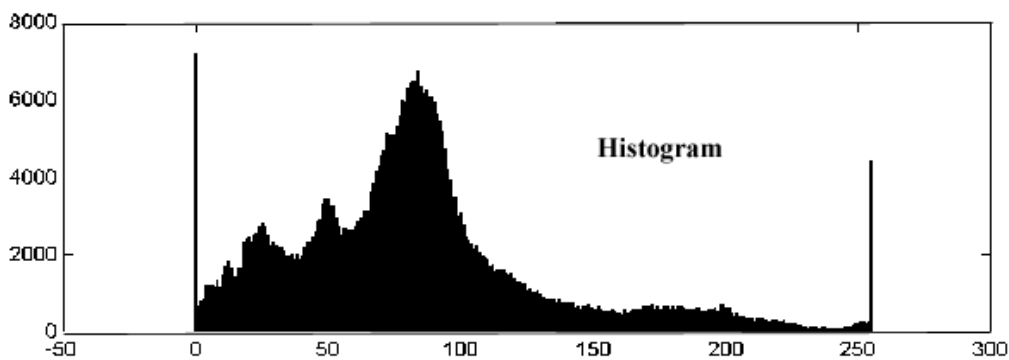


Figure 4: The histogram of intensity corresponding to the image in Figure 3.

4. IMAGE RECORDING

The video-camera used for the project was a Sony CCD camera XC-75CE with an imaging array of 752(H) x 582(V) pixels. This camera was operated in what is referred to as Mode 2 (a stop-restart mode) which allowed control over both the exposure (shutter) time and the epoch at which the frame was grabbed. The exposure time used was $1/10,000^{\text{th}}$ sec. Faster exposure times are possible with an external signal but this shutter speed was considered adequate since the maximum frequency of excitation examined was 17Hz, which corresponded to a maximum phase error of

0.6°. Due to idiosyncrasies of the camera in the start-stop mode, only the second frame contained useful information, the first field of the second frame being recorded after an additional delay of 12.5 milliseconds. This additional delay was on top of any desired delay. The source of illumination used was a high powered flash which dissipated 6 kilo-joules in 20 milliseconds. This guaranteed that good contrast was possible even for such short exposure times.

In the actual image analysis, video fields rather than frames were employed (since the fields are taken at different times and separated by 1/50th sec) which meant that ‘vertical resolution (number of image rows) was effectively half that of the horizontal (number of columns). A preliminary investigation indicated that the use of video fields alone would have only a minor impact on the precision of measuring target centroids within the imagery. However, since it was known *a priori* that the error in the *y* coordinate would likely be twice that in *x*, the optimal way of treating the data would have been through use of a weighted least squares approach where the *x*-coordinate observations (‘horizontal’) would be assigned twice the weight of those in *y* (‘vertical’). This would yield parameter estimates which were asymptotically unbiased and which would additionally be minimum variance estimators asymptotically. While such a weighting scheme could have been accommodated in *Australis*, indications were that any influence of this basic difference in observational precision between *x* and *y* observations would be minimized by ensuring that the camera was rotated through 90° for every camera position.

An important issue regarding metric quality, which underpinned the whole method, was that the shaker would produce perfectly repeatable oscillation cycles. This was tested by recording multiple images at different times, when the deformed wing was nominally in the same state. The image coordinates were then measured in each image and were found to be repeatable to within 0.05 pixels. The assumption of shape invariance for a given epoch had therefore been experimentally verified.

5. INITIAL VALIDATION OF THE METHOD

An initial test was carried out to validate the off-line, single-sensor vision metrology approach for measuring the vibrating wing section. This involved a small metal plate with six attached targets being rigidly attached to a shaker, as shown in Figure 5. As a result, all motion induced by the shaker was purely translational in the vertical direction. Moreover, all moving targets would be displaced by the same amount, and since the shaker is a linear device, a reduction by a fixed amount in its driving voltage should reduce the motion by the same amount. All other targets indicated in Figure 5 were stationary so as provide a fixed *XYZ* reference coordinate system.

Table 1 shows the peak-to-peak displacements during a cycle of oscillation experienced by the targets labelled 1 and 2, as measured by the off-line photogrammetric approach employing 8 camera stations. The distance between targets 2 and 6 is equal to 2.5cm, this distance being used as the reference scale. For a fixed gain the points 1 and 2 moved through distances that differed by less than the anticipated measurement tolerance, less than 0.05mm in this case. Also shown in the table are normalised displacements for the two points at each nominal gain and it can

be seen that the estimated relative displacements are equal to (within a very small error) the relative voltage gains driving the shaker.

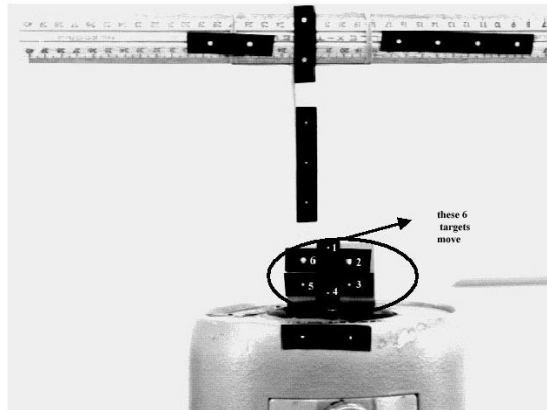


Figure 5: Target configuration for validation test; targets 1-6 are mounted on a shaker and the remainder are stable.

Table 1. Absolute and relative displacements of points 1 and 2 within the validation test measurements.

Gain	Displacement at Pt.1 (absolute, μm) (relative)		Displacement at Pt.2 (absolute, μm) (relative)	
1	414	1	345	1
1/2	234	0.56	176	0.51
1/3	138	0.33	119	0.34
1/4	96	0.23	89	0.26

6. EXPERIMENTAL METHOD AND RESULTS

As indicated in Figure 2, the structure to be examined was a wing section held in a supporting frame. Also shown in the figure is a strobe lamp mounted behind the camera to provide on-axis illumination for the retroreflective targets forming the object point array. Oscillation in the wing structure was induced by the shaker shown in the foreground and the camera, flash and shaker were all controlled with reference to a computer generated sinusoidal wave. As mentioned, adjustments to the time delay between the primary reference signal and the camera ensured that spatial positions within the cycle of oscillation were captured at various phases of motion. The use of only one phase might well have resulted in the capture of motion where the deviation from the mean or static position was very close to zero, thus making the estimate of mode shape unreliable. As a consequence, the time delays used in this experiment were $T = 0, 1/4f, 2/4f$ and $3/4f$, where f is the frequency of excitation. These delays corresponding to phase shifts of $0^\circ, 90^\circ, 180^\circ$ and 270° . The selection of four phases both eliminated the problem of minimal deviation and provided a level of

redundancy. Static measurements corresponding to absolutely no motion were also performed. This information defined where the targets were positioned in space as distinct from describing the additional differential displacements due to motion.

A total of eight different camera positions were used for each photogrammetric survey, and two modes were studied. These modes corresponded to the fundamental bending mode and the torsion mode of vibration, which occurred at 11Hz and 17Hz, respectively. As mentioned above, four phases were captured at each frequency. As a result, for each oscillation test there were 8 independent spatial triangulation determinations (via Eq. 4) and a final 64-image bundle adjustment for the simultaneous determination of all photogrammetric parameters. A notable benefit of the final bundle adjustment is that all XYZ object points are explicitly determined in a common reference coordinate system which is nominally based on the 12 stable targets positioned off the wing (see Figure 2). As it happened, the exterior orientation parameters did remain very stable over time and there was therefore no significant difference in the XYZ coordinates obtained via spatial intersection alone (Eq. 4) and those computed within the full bundle adjustment (Eq. 2).

As previously mentioned, a standard ‘static’ measurement of the wing at rest was also carried out via bundle adjustment prior to each oscillation cycle, though this was not strictly necessary as the same information should also be contained within the positional data obtained for the various phases. Use of the static information, along with the four different phases, enabled a mono-phase mode to be calculated. The mono-phase mode is defined as one in which there is no phase change with position except for a possible sign change. This mode can thus be represented by using only real numbers or real functions. If the position of the i th target at time t is given by $R_i \cos \theta \cos \omega t + R_i \sin \theta \sin \omega t$, then a least squares estimation process can be used to determine θ . Although this least squares problem is nonlinear an analytical solution can be formulated, though some care needs to be taken in selecting the correct value of θ as multiple solutions are involved. Subsequently, R_i can be determined in a linear manner.

Figures 6 and 7 illustrate the final results for the first two modes of vibration of the “wing and rig structure”. The plot scale of the point displacement vectors $(0,0,R_i)$ in the figures has been exaggerated to better illustrate the pattern of motion. It should be noted that the system is not a pure cantilever as the nodal lines (lines of no motion) running from wing-leading edge to wing-trailing edge do not occur at the position of attachment of the wing to the rig. Aside from this, the accuracy of determining the modes of vibration was in accordance with design expectations (measurement to 0.05mm accuracy) and the shape of both modes corresponds to expectations for such a simple structure.

7. CONCLUSION

In this paper it has been shown that an off-line, single-sensor vision metrology approach was successfully employed to determine the dynamic modes of a vibrating structure. Initially a simple experiment (a rigid plate and shaker where the shape of vibration was a priori known) checked the viability of the procedure as well as providing error estimates. A more complex structure involving a wing section was

also studied and in particular its first two modes of vibration were determined and a discussion of errors was also made. The method used has the advantage of being a low cost non-intrusive method of determining dynamic responses from structures where the excitation is able to be controlled.

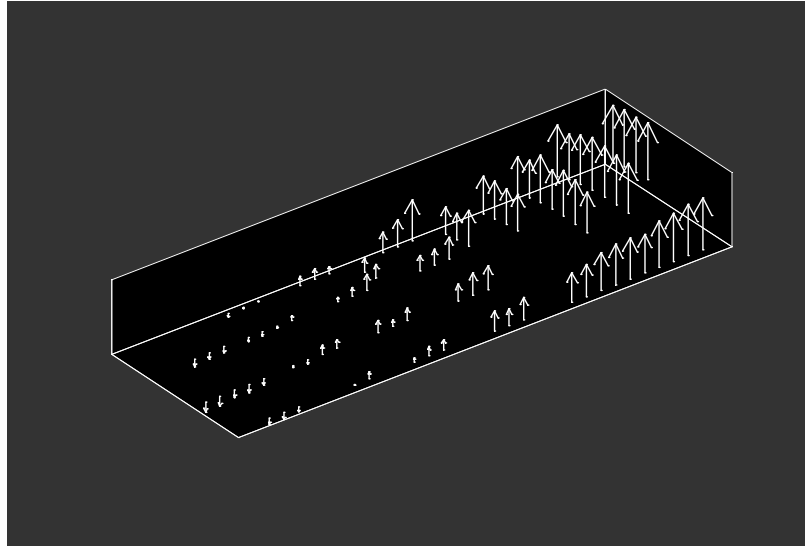


Figure 6: First Mode, Fundamental Bending

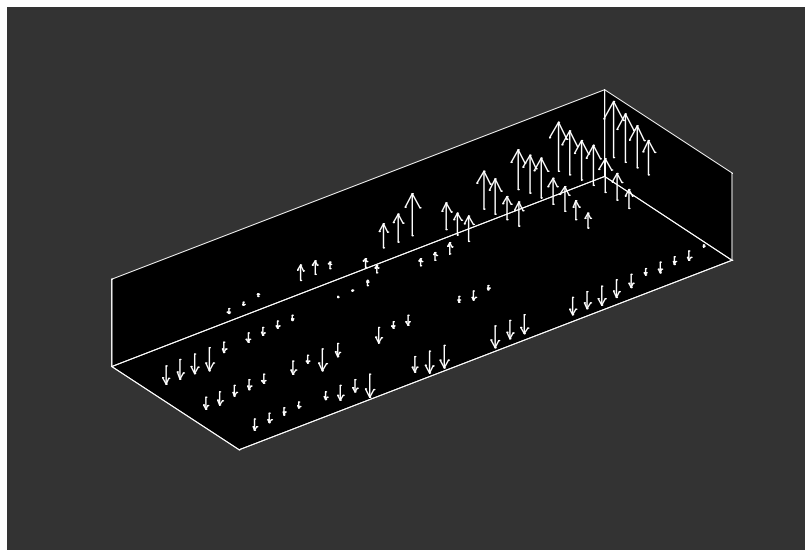


Figure 7: Second Mode, Torsion Mode

8. ACKNOWLEDGMENTS

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